QED radiative corrections to the decay $\pi^0 \to e^+e^-$

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Abstract

We reconsider QED radiative corrections (RC) to the $\pi^0 \to e^+e^-$ decay width. One kind of RC investigated earlier has a renormalization group origin and can be associated with the final state interaction of electron and positron. It determines the distribution of lepton pair invariant masses in the whole kinematic region. The other type of RC has a double-logarithmic character and is related to almost on-mass-shell behavior of the lepton form factors. The total effect of RC for the $\pi^0 \to e^+e^-$ decay is estimated to be 3.2% and for the decay $\eta \to e^+e^-$ is 4.3%.

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I. INTRODUCTION

Rare decays of mesons serve as the low-energy test of the Standard Model. Accuracy of experiments has increased significantly in recent years. Theoretically, the main limitation comes from the large distance contributions of the strong sector of the Standard Model where the perturbative theory does not work. However, in some important cases the result can be essentially improved by relating these poorly known contributions to other experimentally known processes. The famous example is the Standard Model calculation of the anomalous magnetic moment of muon $(g-2)_\mu$ where the data of the processes $e^+e^- \to hadrons$ and $\tau \to hadrons$ are essential to reduce the uncertainty. It turns out that this is also the case for the rare neutral pion decay into an electron-positron pair measured recently by the KTeV collaboration [1] and reconsidered theoretically in [2].

The measured branching is [1]

$$B^{\rm KTeV}\left(\pi^0 \to e^+e^-, x_D > 0.95\right) = (6.44 \pm 0.25 \pm 0.22) \cdot 10^{-8}, \tag{1}$$

where the kinematic cut over the Dalitz variable $x_D \equiv (p_+ + p_-)^2/M^2$, $\nu^2 \equiv 4m^2/M^2 \leq x_D \leq 1$, was used in order to suppress the Dalitz decay events $\pi^0 \to e^+e^-\gamma$. Then, the important step in extraction of the branching consists in correct treating the radiative corrections (RC) to the process which has been considered earlier in [3] and [11]. Extrapolating the full radiative tail beyond $x_D > 0.95$ and scaling the result back up by the overall RC leads to the final result [1]

$$B_0^{\rm KTeV}(\pi_0 \to e^+e^-) = (7.49 \pm 0.29 \pm 0.25) \cdot 10^{-8}, \tag{2}$$

where the leading order radiative corrections have been taken into account [3]. It is the motivation of our paper to revise the calculation of QED RC to the $\pi_0 \to e^+e^-$ decay width.

In the lowest order of QED perturbation theory (PT), the photonless decay of the neutral pion,

$$\pi_0(q) \to e^-(p_-) + e^+(p_+), \quad q^2 = M^2, \quad p_\pm^2 = m^2,$$

(M meson mass, m lepton mass) is described by the one-loop Feynman amplitude (Fig. 1a) corresponding to the conversion of the pion through two virtual photons into an electron-positron pair. The normalized branching ratio is given by [4, 5, 6]

$$R_0(\pi_0 \to e^+e^-) = \frac{B_0\left(\pi^0 \to e^+e^-\right)}{B\left(\pi^0 \to \gamma\gamma\right)} = 2\beta\left(\frac{\alpha m}{\pi M}\right)^2 |\mathcal{A}\left(M^2\right)|^2, \tag{3}$$

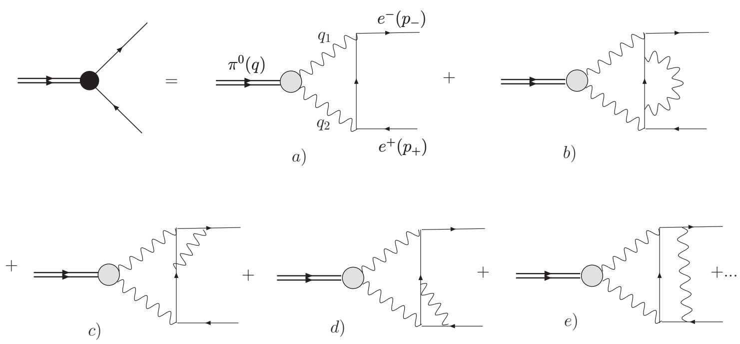

FIG. 1: Set of the lowest order QED RC to $\pi^0 \to e^+e^-$ process: virtual corrections.

where $\beta = \sqrt{1-\nu^2}$, $B\left(\pi^0 \to \gamma\gamma\right) = 0.988$ and the reduced amplitude is

$$\mathcal{A}\left(q^2\right) = \frac{2}{M^2} \int \frac{d^4k}{i\pi^2} \frac{(qk)^2 - q^2k^2}{(k^2+i\epsilon)\left[(q-k)^2+i\epsilon\right]\left[(p_- - k)^2 - m^2 + i\epsilon\right]} F_\pi(-k^2, -(q-k)^2), \quad (4)$$

with the pion transition form factor $F_\pi(-k^2, -q^2)$ being normalized as $F_\pi(0,0) = 1$. The imaginary part of $\mathcal{A}\left(q^2\right)$ can be found in a model independent way [5]

$$\mathrm{Im}\mathcal{A}(M^2) = -\frac{\pi}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right), \quad (5)$$

while the real part is reconstructed by using the dispersion approach up to a subtraction constant

$$\mathrm{Re}\mathcal{A}(M^2) = \mathcal{A}(0) + \frac{1}{\beta}\left[\frac{\pi^2}{12} + \frac{1}{4}\ln^2\left(\frac{1+\beta}{1-\beta}\right)\right]. \quad (6)$$

Usually this constant, containing the nontrivial dynamics of the process, is calculated within different models describing the form factor $F_\pi(k^2, q^2)$ [2, 6, 7, 8]. However, it has recently been shown in [2] that this constant may be expressed in terms of the inverse moment of the pion transition form factor given in symmetric kinematics of spacelike photons

$$\mathcal{A}\left(q^2=0\right) = 3\ln\left(\frac{m_e}{\mu}\right) - \frac{3}{2}\left[\int_0^{\mu^2} dt \frac{F_{\pi\gamma^*\gamma^*}(t,t) - 1}{t} + \int_{\mu^2}^{\infty} dt \frac{F_{\pi\gamma^*\gamma^*}(t,t)}{t}\right] - \frac{5}{4}. \quad (7)$$

Here, μ is an arbitrary (factorization) scale. One has to note that the logarithmic dependence of the first term on μ is compensated by the scale dependence of the integrals in the brackets. The accuracy of these calculations are determined by omitted small power corrections of the order $O(\frac{m^2}{\Lambda^2})$ and $O(\frac{m^2}{M^2}L)$ in the r.h.s. (6), where $\Lambda \lesssim M_\rho$ is the characteristic scale of the form factor $F_{\pi\gamma^*\gamma^*}(t,t)$ and L is the large logarithm parameter

$$L = \ln\left(\frac{M^2}{m^2}\right) \approx \ln\left(\frac{1+\beta}{1-\beta}\right).$$

For the decay $\pi^0 \to e^+e^-$ one has $L \approx 11.2$.

By using the representation (7), and the CELLO [9] and CLEO [10] data on the pion transition form factor $F^{\rm CLEO}_{\pi\gamma^*\gamma^*}(t,0)$ given in asymmetric kinematics the lower bound on the decay branching ratio was found in [2]. This lower bound follows from the property: $F_{\pi\gamma^*\gamma^*}(t,t) < F_{\pi\gamma^*\gamma^*}(t,0)$ for $t > 0$. It considerably improves the so-called unitary bound obtained from the property $|\mathcal{A}|^2 \geq (\mathrm{Im}\,\mathcal{A})^2$. Further restrictions follow from QCD and allow one to make a model independent prediction for the branching [2]

$$B_0^{\rm Theor}\left(\pi^0 \to e^+e^-\right) = (6.2 \pm 0.1) \cdot 10^{-8}, \tag{8}$$

which is 3.3σ below the KTeV result (2). The main source of the error in (8) is defined by indefiniteness in the knowledge of the pion form factor $F_{\pi\gamma^*\gamma^*}(t,t)$ [2]. The discrepancy between (8) and (2) requires further attention from experiment and theory to this process because there are not many places where experiment is in conflict with the Standard Model.

Considering the higher orders of QED PT, there are two sources of RC to the width of the $\pi^0 \to e^+e^-$ decay (Fig. 1). One of them has a renormalization group origin and can be associated with the final state interaction of electron and positron. The relevant contribution corresponds to taking into account the charged particle interaction at large distances. The other is of double-logarithmic character and is related to short distance contributions. Let us note that the branching ratio (3) is proportional to the electron mass squared and thus the Kinoshita–Lee-Nauenberg theorem of cancellation of mass singularities in the limit $m \to 0$ is not violated.

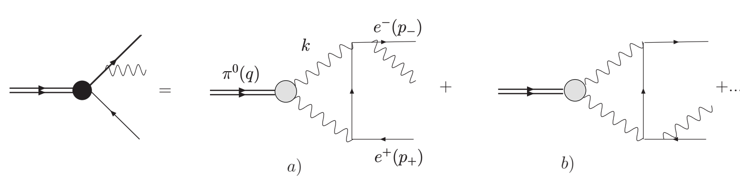

FIG. 2: RC due to soft photon emission.

The first kind of corrections was considered in [3]. Later, the effect of the higher order RC was estimated in [11] by using the exponentiation of soft photon contributions, which is essentially equivalent to the Yennie, Frautchi-Suura factorization procedure. Considering only the two-virtual-photon conversion to a lepton pair, it is originated from the box type Feynman amplitude (Fig. 1e) as well as the contribution from the emission of real photons

by leptons (Fig. 2) and produces the single-logarithmic enhanced terms ($\sim L$) which are described by the lepton nonsinglet structure function method [12]. It was shown in [11] that the soft photon emission can drastically change the results obtained at the Born level when the invariant mass of leptons is close to the pion mass.

Furthermore, we find an additional source of RC which is of the so-called "double-logarithmic" (DL) nature ($\alpha L^2/\pi \sim 1$). This kind of asymptotics was intensively investigated in the 70s in a series of QED processes [13, 14]. The DL type contribution to the decay width was not considered earlier for the $\pi^0 \to e^+e^-$ decay.

II. LARGE LOGARITHM REGIME AND DOUBLE-LOGARITHMIC CORRECTION

First, it is instructive to reproduce the results discussed above in a simple and physical way. For this aim we note that the main contribution to the real part of $\mathcal{A}\left(M^2\right)$ comes from the kinematic region of loop momenta corresponding to the intermediate virtual electron (or positron) close to the mass shell (Fig. 1a). Really, by changing the integration variable in (1) as $q_1 = k = p_- - \kappa, \quad q_2 = q - k = p_+ + \kappa$ and omitting terms of order $O(\kappa^2/M^2)$ we can rewrite the amplitude in the Born approximation $\mathcal{A}_0\left(M^2\right)$ as

$$\mathcal{A}_0\left(M^2\right) \approx \frac{1}{2}\int \frac{d^4\kappa}{i\pi^2} \frac{M^2}{(\kappa^2 - m^2 + i\epsilon)((\kappa - p_-)^2 + i\epsilon)((\kappa + p_+)^2 + i\epsilon)}.$$

Let us find the real part of the amplitude within the leading logarithmic accuracy that corresponds to the restriction of the kinematic region by conditions

$$m^2 \approx |\kappa|^2 \ll \left(|q_1^2|, |q_2^2|\right) \ll M^2. \tag{9}$$

To this end one performs the substitutions

$$\begin{aligned} &\frac{d^4\kappa}{\Delta + i\epsilon} \to \frac{-i\pi \overrightarrow{\kappa}^2 d|\overrightarrow{\kappa}| dO_\kappa}{2\omega} \left[\Theta\left(\kappa_0\right) + \Theta\left(-\kappa_0\right)\right]|_{\kappa_0^2 = \omega^2}, \\ &q_1^2 = -2M\kappa_0 u, \quad q_2^2 = 2M\kappa_0\left(u + \beta_\omega \cos\theta\right), \\ &\omega = \sqrt{\overrightarrow{\kappa}^2 + m^2}, \quad \Delta = \kappa^2 - m^2, \quad \beta_\omega = \sqrt{1 - \frac{m^2}{\omega^2}}, \qquad u = \frac{1 - \beta_\omega \cos\theta}{2} \end{aligned} \tag{10}$$

where θ is the angle between the directions of electron momentum (the rest frame of the initial pion implied) and the 3-momentum of the virtual electron. Let us note that by

kinematical reasons in the region of maximal contribution the signs of q_1^2 and q_2^2 must be opposite. Performing the angular integration we obtain the leading term of (6)

$$\mathrm{Re}\mathcal{A}_0\left(M^2\right) = -\frac{M^2}{\pi}\int\frac{d^3\kappa}{\omega q_1^2 q_2^2} = \int_m^{\frac{M}{2}}\frac{\beta_\omega d\omega}{\omega}\ln\frac{\omega^2}{m^2} \approx \frac{1}{4}L^2. \tag{11}$$

Then, let us consider the vertex type RC (Fig. 1 c,d). The lowest order evaluation arising from the diagrams of Figs. 1b) and 1c) leads to the correction $\Gamma(q_1,\kappa,p_-) = 1 - \frac{\alpha}{2\pi}I_V(q_1^2,\kappa^2)$ with

$$I_V(q_1^2,\kappa^2) = \ln\frac{|q_1^2|}{m^2}\ln\frac{|q_1^2|}{|\kappa^2|} + \frac{1}{2}\ln^2\frac{|q_1^2|}{|\kappa^2|} - \frac{3}{2}\ln\frac{|q_1^2|}{m^2} + \frac{\pi^2}{3} + \frac{1}{2} - \ln\frac{m}{\lambda} - \Theta\left(-q_1^2\right)\frac{3\pi^2}{2},$$

which is consistent with the result of similar calculations in [15]. The last term arises from a renormalization procedure. A similar contribution $I_V(q_2^2,\kappa^2)$ comes from the diagrams of Figs. 1b) and 1d).

Let us consider now the box-type diagram (see Fig. 1e) and demonstrate the calculations in more detail. The corresponding contribution to the amplitude has the form

$$\frac{\Delta}{2}\,\mathrm{Re}\int\frac{d^4k_1}{i\pi^2}\frac{N}{(k_1^2-\lambda^2+i\epsilon)\left((p_-+k_1)^2-m^2+i\epsilon\right)\left((p_+-k_1)^2-m^2+i\epsilon\right)\left((\kappa+k_1)^2-m^2+i\epsilon\right)}, \tag{12}$$

where

$$N = \bar{u}(p_-)\gamma_\mu(p_-+k_1+m)\gamma_\lambda(\kappa+k_1+m)\gamma_\gamma(-p_++k_1+m)\gamma_\mu v(p_+).$$

In the leading kinematic region, where k_1 and κ are small, we can reduce the numerator to

$$N \approx -2mM^2\bar{u}(p_-)\gamma_\lambda\gamma_\gamma v(p_+).$$

The calculation of the scalar 4-denominator integral is standard: by using the Feynman parametrization and performing loop momentum integration we arrive at

$$-2mM^2\Delta\int_0^1 dx\int_0^1 y dy\int_0^1\frac{z^2dz}{\left(Az^2+Bz+C\right)^2},$$

with

$$A = (yp_x - \bar{y}\kappa)^2;\ B = -\bar{y}\Delta - \lambda^2;\ C = \lambda^2, \tag{13}$$

$$p_x = xp_+ - \bar{x}p_-,\quad p_x^2 = m^2 - M^2x\bar{x},\quad \bar{x} = 1-x,\ \bar{y} = 1-y.$$

First, we perform the integration in z

$$\int_0^1 \frac{z^2 dz}{(Az^2+Bz+C)^2} = \frac{2C+B}{(A+B+C)R} - \frac{2C}{R^{\frac{3}{2}}} \ln \frac{2C+B+\sqrt{R}}{2C+B-\sqrt{R}}, \tag{14}$$
$$R = B^2 - 4AC > 0.$$

Calculating the y-integral of (14) results in

$$-\frac{1}{p_x^2}\left[\frac{1}{2}\ln\frac{\Delta^2}{\lambda^2 m^2} + \frac{1}{2}\ln\frac{p_x^2}{m^2} - \ln\left|\frac{p_x^2}{q_1^2\bar{x}+q_2^2 x}\right|\right].$$

Integration in x by using

$$\mathrm{Re}\int_0^1 \frac{dx}{p_x^2+i\epsilon} = -\frac{2}{M^2}L, \quad \mathrm{Re}\int_0^1 \frac{dx}{p_x^2+i\epsilon}\ln\frac{p_x^2+i\epsilon}{m^2} = -\frac{1}{M^2}(L^2-\frac{4}{3}\pi^2),$$

leads to the correction $\frac{\alpha}{2\pi}I_B$ with

$$I_B\left(q_1^2,q_2^2\right) = -\frac{1}{2}L^2 - 2(L-1)\ln\frac{m}{\lambda} - L(L_1+L_2) - \frac{1}{2}(L_1-L_2)^2 + \frac{1}{2}\pi^2, \tag{15}$$

where

$$L_{1,2} = \ln\frac{|q_{1,2}^2|}{m^2}.$$

Finally, one needs to integrate over photon momenta q_1 and q_2. Again, the logarithmically enhanced contribution comes from the kinematic regions $m \le \omega \le M/2$, $\cos\theta \to \pm 1$ (see definitions in (10)). The Born amplitude (one-loop) and the lowest-order radiative correction to it can be written as (we take into account the equal contributions of regions $|q_1^2| \ll |q_2^2|$ and $|q_2^2| \ll |q_1^2|$)

$$\sim \int_m^{\frac{M}{2}} \frac{d\omega}{\omega}\beta_\omega \int_{\frac{m^2}{4\omega^2}}^1 \frac{du}{u}\left[1 + \frac{\alpha}{2\pi}(I_V\left(q_1^2,m^2\right) + I_V\left(q_2^2,m^2\right) + I_B\left(q_1^2,q_2^2\right))\right] \tag{16}$$

with (we put here $\Delta \approx m^2$)

$$\begin{aligned} I_V\left(q_1^2,m^2\right) + I_V\left(q_2^2,m^2\right) + I_B\left(q_1^2,q_2^2\right) &= -\frac{1}{4}L^2 - \frac{1}{2}Ll - \frac{3}{4}l^2 - 2(L-1)\ln\frac{m}{\lambda} + \frac{3}{2}L + \frac{3}{2}l \\ &- l_u^2 - \left(\frac{1}{2}L + \frac{3}{2}(l-1)\right)l_u + \frac{\pi^2}{6} + 1, \end{aligned} \tag{17}$$

where we use substitutions (10) and introduce the notation

$$l = \ln\left(\frac{4\omega^2}{m^2}\right), \qquad l_u = \ln\left(\frac{1-\beta_\omega\cos\theta}{2}\right). \tag{18}$$

Integration of (16) leads to (we keep only terms of order L^2, L and L^0)

$$\frac{R_{virt}}{R_0} = 1 + \delta_{virt},$$
$$\delta_{virt} = \frac{\alpha}{\pi}\left[-\frac{13}{24}L^2 - 2(L-1)\ln\frac{m}{\lambda} + \frac{3}{4}L + \frac{\pi^2}{6} + 2\right]. \tag{19}$$

Consider now the real photon emission corrections. One can distinguish two mechanisms of the radiative decay $\pi^0 \to e^+e^-\gamma$. One of them, the so-called Dalitz process, corresponds to decay mode of the pion to real and virtual photons with a subsequent decay of the virtual photon to the e^+e^- pair. The corresponding contribution to the width is not suppressed by lepton mass and provides an important background to the $\pi_0 \to e^+e^-$ process [3, 16, 17]. However, the Dalitz matrix element squared and its interference with the double virtual photon amplitude are suppressed by $\sim (1-x_D)^3$ and $\sim (1-x_D)^2$ as $x_D \to 1$ [3] (Fig. 3). This results in a negligible (of order 0.02%) interference contribution integrated in the region of interest for this measurement, $0.9 < x_D < 1$ [19].

Another mechanism consists in creation of a lepton pair by two virtual photons with emission of real photon by a pair components. For emission of a soft photon (with energy ω not exceeding $\Delta\epsilon << \frac{M}{2}$ in the pion rest frame) the standard calculations [13] give

$$\delta_{\rm soft} = \frac{\alpha}{\pi}\left[2\left(L-1\right)\ln\frac{2\Delta\varepsilon}{M} + 2\left(L-1\right)\ln\frac{m}{\lambda} + \frac{1}{2}L^2 - \frac{\pi^2}{3}\right]. \tag{20}$$

Emission of a hard photon was investigated in [3] with the result

$$\delta_{\rm hard} = \frac{\alpha}{\pi}\left[-2\left(L-1\right)\ln\frac{2\Delta\varepsilon}{M} - \frac{3}{2}\left(L-1\right) - \frac{\pi^2}{3} + \frac{7}{4}\right]. \tag{21}$$

In order to find the distribution over a lepton pair invariant mass, the adequate way is to use the method of structure functions [12] based on the application of the renormalization group approach to QED. Here, the nonsinglet structure function is associated with a final state fermion line. In partonic language it describes the probability for a fermion to stay a fermion. Omitting the events with creation of more than one lepton, the nonsinglet structure function of electron relevant to the process valid at all orders in perturbation theory is[1]

$$F(x_D) = b\left(1-x_D\right)^{b-1}\left(1 + \frac{3}{4}b\right) - \frac{1}{2}b(1+x_D) + O\left(b^2\right), \tag{22}$$

[1] Note that the value of the K-factor in (22), $K = 1 + \frac{3}{4}\frac{\alpha}{\pi}$, differs from that obtained in [12] for the process of hadron production in single-photon e^+e^- annihilation channel.

where $b = \frac{2\alpha}{\pi}(L-1)$. To the lowest order in b and in the region $x_D >> \nu^2 = 4m^2/M^2$, the above expression is in agreement with the leading order expression [3]

$$\frac{1}{R_0}\frac{dR_{\rm LO}^{\rm brem}(x_D)}{dx_D} = \frac{\alpha}{\pi}\frac{1}{1-x_D}\left\{(1+x_D^2)\ln\left(\frac{1+\beta_x}{1-\beta_x}\right) - 2x_D\beta_x\right\}, \tag{23}$$

where

$$\beta_x = \sqrt{1-\frac{\nu^2}{x_D}}.$$

Thus we arrive at the differential rate

$$\frac{1}{R_0}\frac{dR_\pi^{RC}(x_D)}{dx_D} = JF(x_D), \tag{24}$$

where the normalization factor J takes into account to total RC. The distribution (24), shown in Fig. 3, in contrast to (23) is free of nonintegrable singularity at $x_D = 1$. So, we see the importance of taking into account the higher orders of perturbation theory.

Adding the virtual and real photon emission contributions we finally obtain

$$\begin{aligned}\frac{R_\pi^{RC}}{R_0} &= J\int_0^1 F(x_D, L)dx_D = 1 + \alpha_{virt} + \alpha_{soft} + \alpha_{hard} \\ &= 1 - \frac{\alpha}{\pi}\left[\frac{1}{24}L^2 + \frac{3}{4}L - \frac{\pi^2}{2} + \frac{21}{4}\right] \approx 0.968.\end{aligned} \tag{25}$$

The total radiative correction contains the large logarithm term $\sim L$ because the γ_5 current is not conserved. Compared with [3] we provide a more detailed analysis of the effective $\pi^0 \to e^+e^-$ vertex revealing its DL structure[2]. However, numerically the ratio of the total RC corrections to the lowest level rate estimated in (25) is very close to -3.4% found in [3] and used by the KTeV Collaboration in their analysis.

The branching ratio of the pion decay into an electron-positron pair has been measured by the KTeV collaboration in the restricted kinematic region in order to avoid a large background from the Dalitz process dominating at lower values x_D. By using the distribution (22) we can estimate the factor of extrapolation of the full radiative tail beyond $x_D > 0.95$ as $f_{0.95} = 1.114$. With this factor and scaling the result back up by the overall radiative correction (25) we confirm the result (2) obtained by the KTeV Collaboration.

[2] It is naturally to expect the existence of DL-type RC in higher orders of perturbative theory. We do not touch this problem here.

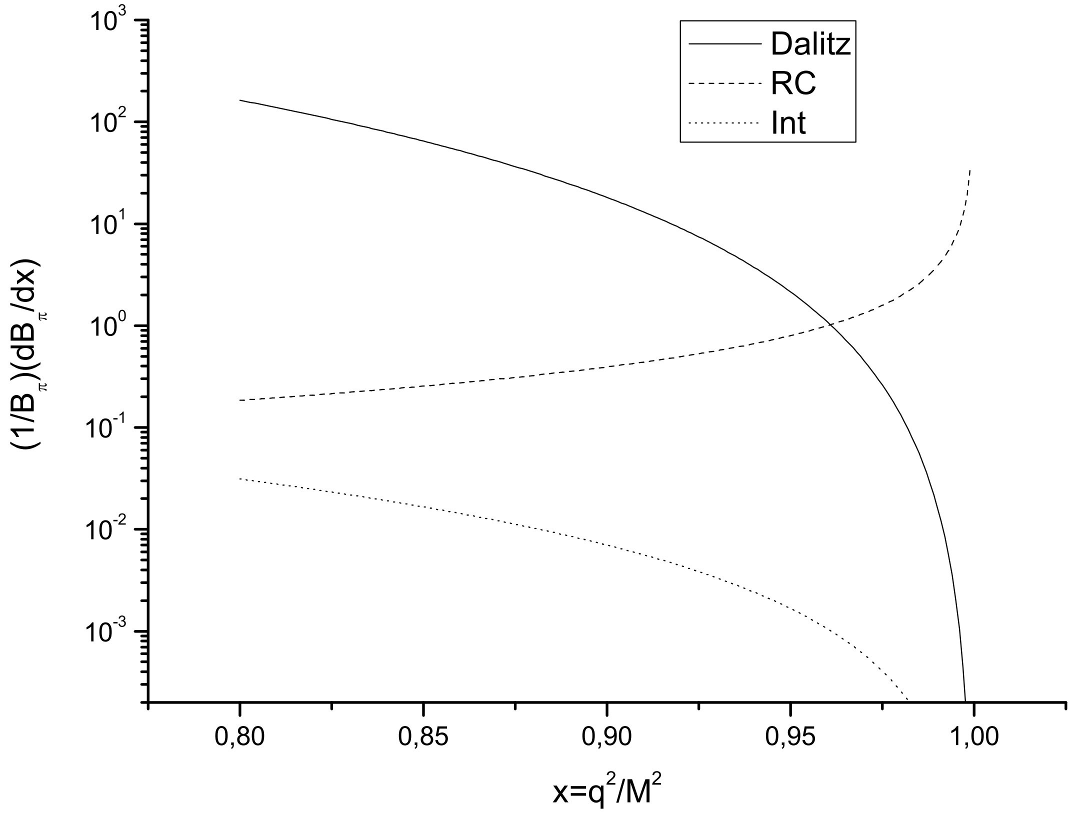

FIG. 3: Distribution of different contributions over $x = \frac{M_{ee}^2}{M^2}$: solid line - Dalitz mechanism contribution, dashed line - inner bremsstrahlung contribution (24), dotted line - interference of these two mechanisms.

III. CONCLUSIONS

In this work, we reconsidered the contribution of QED radiative corrections to the $\pi^0 \to e^+e^-$ decay which must be taken into account when comparing the theoretical prediction (8) with experimental result (1). Comparing with earlier calculations [3], the main progress is in detailed consideration of the $\gamma^*\gamma^* \to e^+e^-$ subprocess and revealing of dynamics of large and small distances. The large distance subprocess associated with final state interaction produces the terms linear in the large logarithm parameter L. The double logarithmic contributions ($\sim L^2$) correspond to configurations when the particles in the loop are highly virtual. The total result is in reduction of the normalization factor by $1 - J \approx 0.032$.

Occasionally, this number agrees well with the earlier prediction based on calculations [3] and thus we confirm the KTeV analysis of RC factors. So our main conclusion is that taking into account of radiative corrections is unable to reduce the discrepancy between the theoretical prediction for the decay rate (8) and experimental result (2). Further independent experimental and theoretical efforts are necessary. Note, that if the discrepancy will stand as it is, then the possible explanation of the effect is due the contribution to the decay width of low mass ($\sim$ 10 MeV) vector boson appearing in some models of dark matter [21].

The question about the corrections is also important for the decays of η and K mesons to $\mu^+\mu^-$ and must be taken into account for the analysis of experimental data. However, for these decays the large logarithm parameter does not arise. The analysis of the lowest order RC to the decay width of kaon to muon pair was given in [18]. Unfortunately, the result of [18] explicitly depends on infrared singularities and cannot be used in practice. Thus, in order to extract the information about $P \to l\bar{l}$ decays (where $P = K_L, \pi_0, \eta, \ldots$ and $l = e, \mu$) calculated within the frame of definite models, the RC must be taken into account when working with experimental data, e.g., for the process $\eta \to \mu^+\mu^-$ [20]. For the process $\eta \to e^+e^-$ our results are applicable and we estimate RC at the level 4.3%.

Finally, we have to emphasize that the role of RC is rather timely because of the growing accuracy of modern experiments.

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